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Some observations of the transition process on the windward face of a long yawed cylinder

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An experiment has been performed to determine the effect of yaw upon transition in the boundary layer formed on the windward face of a long cylinder. The china-clay-evaporation and surface-oil-flow techniques have been used to study the development of the fixed-wavelength stationary disturbances which are characteristic of cross-flow instability. It has been found that the boundary layer is also susceptible to time-dependent disturbances which grow to very large amplitudes prior to the onset of transition. These disturbances have been studied with a hot-wire anemometer. The conditions necessary for the onset and completion of transition have been determined by the use of surface Pitot tubes. Data from the experiment have been compared with the simple criteria for instability and transition which were proposed by Owen & Randall over thirty years ago. In general it has been found that these criteria are inadequate, and, where possible, improvements have been proposed. The raw data are presented in sufficient detail for them to be used to test, or calibrate, future theoretical models of the transition process in three-dimensional boundary-layer flows.

1. Introduction

The study of flow about cylinders of general cross-sectional shape is an area of research which provides information of both fundamental and practical importance. Perhaps the most widely studied flow is that about a circular cylinder. This provides a fine example of a situation in which viscous and inviscid regions interact strongly to produce large forces. One of the most striking features of this interaction process is the abrupt change in the flow-field character that takes place when the separating laminar boundary layer reattaches to form a short bubble, or when transition occurs in the attached flow on the windward face and the laminar separation is suppressed. In both these instances a new turbulent separation is established on the leeward face, and there are major changes in the pressure distribution, drag force, lift force and vortex-shedding frequency. Since these changes are produced by a mechanism associated with the viscous flow, they occur as some 'critical' Reynolds number is exceeded. The circular cylinder has many important engineering applications, and therefore this critical Reynolds number has been the subject of numerous investigations, with particular emphasis being placed upon the influence of free-stream disturbances and surface roughness. Another example of a cylinder flow with important engineering applications is that about an aerofoil section. In this case the maximum lifting performance at high incidence is determined by boundary-layer separation and can be strongly Reynolds-number-dependent. At low incidence, transition in the attached boundary layer has an important influence upon the performance, since turbulent skin friction is responsible for approximately 50 % of the total drag force. In recent years the rising cost of fossil fuel has focused attention upon aerofoil drag, since substantial savings are possible if the boundary layer can be maintained in the laminar state at cruise. Therefore there is currently considerable interest in the problem of boundary-layer transition on aircraft wings operating at high Reynolds number.

The majority of the investigations carried out on circular cylinders and aerofoil sections to determine the effects of Reynolds number on the overall flow characteristics have been performed on configurations whose axes have been set normal to the oncoming flow. Consequently, the transition processes studied have been those relevant to two-dimensional mean boundary-layer flows. However, in many engineering applications cylinder elements are yawed relative to the oncoming flow. This represents a situation in which the mean boundary-layer flow is three-dimensional. Several examples of this may be cited, but perhaps the most obvious is the almost universal use of sweptback wings on aircraft that are designed to operate at high speed. In this case typical yaw angles are of order 30°. However, a review of the currently available literature shows that very few specific investigations have been conducted to determine the effect of yaw angle upon the transition Reynolds number for such flows. Nevertheless, it has been established that at sufficiently high Reynolds number the flow is destabilized by yaw and several simple transition criteria have been proposed. These have been based upon the results of qualitative theoretical studies coupled with data from *ad hoc* experiments. They have never been verified in any general sense, and their level of accuracy cannot be considered satisfactory for practical purposes.

Arguably, the most serious obstacle to improved understanding of the effects of yaw upon transition in the cylinder boundary layer is the absence of a detailed experimental investigation. The work described in the present paper seeks to fill this gap by providing information relating to the general movement of the transition front with yaw angle and Reynolds number and, also, to the development of disturbances in the laminar boundary layer ahead of transition. Rather than providing an end in itself, it is hoped that the data contained in this paper will eventually form the basis of a more complete theoretical treatment of the transition process in general three-dimensional flows.

2. A summary of previous work

The problem of boundary-layer transition on a long yawed cylinder appears to have been first addressed almost 40 years ago in theoretical papers by Jones (1947) and Sears (1948). These authors considered the effect of the spanwise flow on the laminar boundary layer formed on a cylinder of infinite length. They concluded that, if all the physical quantities that describe the flow are invariant in the spanwise coordinate direction and provided that the transition is the result of the amplification of vorticity waves of the type studied theoretically by Tollmien (1931) and Schlichting (1933) and experimentally by Schubauer & Skramstad (1947), then transition would always occur at a given chordwise location x_T at a particular value of the chordwise Reynolds number $U_\infty C/\nu$, where U_∞ and C are measured in a plane perpendicular to the cylinder axis (see figure 1). Therefore Jones and Sears expected that the precipitation of transition at a fixed location on a yawed cylinder would require a greater free-stream speed than in the unyawed case – the ratio of the speeds being equal to the secant of the yaw angle. At that time, however, they were not able to provide any experimental evidence to substantiate their arguments. The first author to question

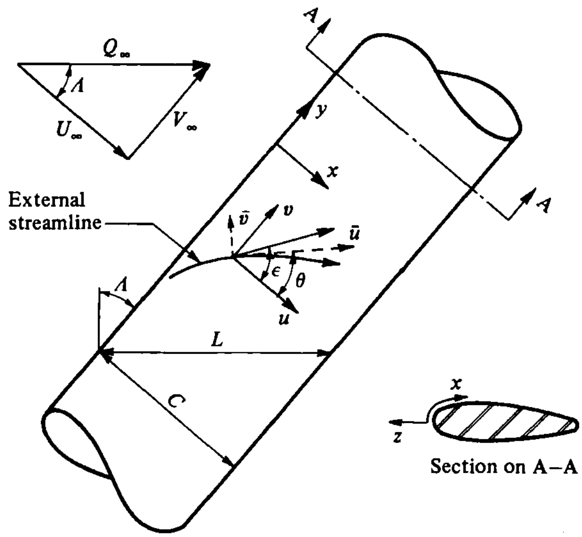


FIGURE 1. Notation and surface coordinate system for an infinite yawed cylinder.

this result was Kuethe (1950), who recognized that a spatially periodic variation of quantities in the spanwise coordinate direction was physically admissible. His subsequent arguments were based upon an analysis for the three-dimensional temporal disturbance waves previously studied by Squire (1933) in connection with the stability of a two-dimensional mean flow confined between parallel walls. Squire had concluded that, for a given wavelength, the disturbance travelling in the mean-flow direction would become unstable at the lowest Reynolds number, i.e. from a stability point of view the most dangerous waves were two-dimensional. However, Kuethe proposed that this would not necessarily be the case if the mean flow was itself three-dimensional. By considering the flow over a flat plate, he demonstrated that for yaw angles between 22.5° and 67.5° a wave whose lines of constant phase made an angle of 45° with the chordwise direction could become unstable at a lower free-stream Reynolds number $Q_\infty L/\nu$ than a 'Jones and Sears' wave whose constant phase lines were perpendicular to the chordwise direction. This led to the conclusion that the 'chordwise' Reynolds number for boundary-layer transition would not be constant, as Jones and Sears had suggested, but would decrease with increasing yaw angle. In an attempt to provide experimental evidence to support this view, Kuethe investigated the effect of yaw upon the critical Reynolds number for a circular cylinder. His measurements, although somewhat limited, appeared to confirm that the chordwise critical Reynolds number was indeed reduced by yaw but the effect was very weak.

Towards the end of the 1940s many experimental aircraft were produced in order to accumulate data on the aerodynamic phenomena encountered during high-speed flight. One of the more unusual configurations to emerge was the 'flying wing', and an example of this type was the Armstrong Whitworth A.W.52. Since the design philosophy behind the flying-wing concept was essentially one of drag reduction, the A.W.52 was provided with wing sections that, in unyawed wind-tunnel tests, had supported laminar flow back to the pressure minimum location at 60% chord. However, despite the fact that the wing was manufactured to exceedingly fine tolerances, flight tests revealed that the boundary layer was turbulent from the

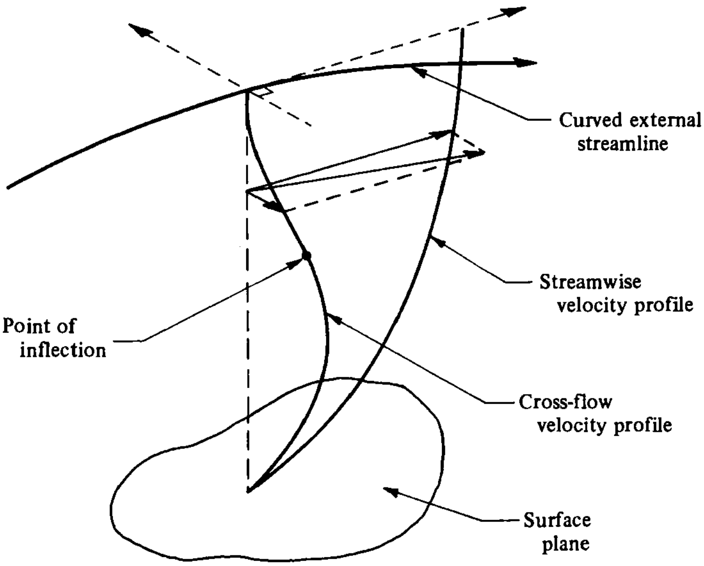


FIGURE 2. Typical boundary-layer velocity distribution near the leading edge of a yawed cylinder.

leading edge on both upper and lower surfaces. This observation was reported by Gray (1952*a*, *b*). Since this was an unexpected result, Gray extended his investigation to cover a range of high-speed swept-wing aircraft. His conclusion from these tests was that no laminar flow was present on conventional (1952) wings of any appreciable size if their sweep exceeded roughly 20°. Gray used the surface sublimation and evaporation techniques to locate transition, and on several of the tests small regions of laminar flow were found near the leading edge. When this occurred, the sublimation pattern in the laminar region sometimes showed a series of closely spaced lines lying almost in the direction of the local flow at the edge of the boundary layer. Gray noted that similar patterns were produced when the same techniques were used to visualize transition in two-dimensional boundary layers on concave surfaces where the closely spaced lines were generated by the action of Görtler vortices. Therefore it was concluded that, at sufficiently high Reynolds number, the effect of yaw (sweep) was to introduce an instability of the laminar flow which resulted in a perturbation taking the form of closely spaced vortices, aligned approximately with the external-flow direction and stationary relative to the solid surface.

The observations reported by Gray clearly indicated that the theoretical work of Jones, Sears and Kuethe was incomplete. Consequently, the problem of the stability of the laminar flow on a yawed infinite cylinder was reconsidered by Squire (1952), Owen & Randall (1952) and Stuart (1952). These authors noted that in the immediate vicinity of the leading edge the streamlines just outside the boundary layer are highly curved in planes drawn parallel to the surface. As the solid surface is approached the retarding action of viscosity results in an inbalance between the centrifugal force per unit volume and the pressure gradient. This produces a secondary flow, or cross-flow, within the boundary layer as shown in figure 2. When viewed in planes perpendicular to the local external streamline, the cross-flow velocity profile is similar to that of a two-dimensional jet issuing into still air, because it has zero velocity at the wall and at the edge of the layer with a maximum velocity being reached at some intermediate location. In view of this, it was argued that such a profile would exhibit instability at a very low Reynolds number, typically of order 100 when based upon maximum cross-flow velocity and viscous-layer thickness. Owen & Randall developed an approximate method for computing the magnitude of this cross-flow Reynolds number χ, and used it to correlate Gray's flight data and also wind-tunnel data from an *ad hoc* experiment performed by Anscombe & Illingworth (1952). It was concluded that the cross-flow Reynolds number was the governing parameter for the transition process and that transition occurred at the chordwise position at which χ was itself a maximum when a value of roughly 175 was reached. The parallel study performed by Stuart (1952) was concerned purely with the theoretical aspects of the instability, as opposed to the question of transition, and, in particular, Stuart addressed the problem of the stationary streamwise streaks in the visualization patterns. Following the experiments of Anscombe & Illingworth and Gregory and Walker in 1952 (see Gregory, Stuart & Walker 1955), it had been recognized that the appearance of these streaks in the visualization patterns was characteristic of the yaw-induced instability. However, it was not clear whether these vortices played an important role in the subsequent transition process. By using classical small-disturbance temporal stability theory Stuart showed that, if the effects of body curvature, streamline curvature and non-parallel flow are negligibly small, then, in a localized region of a three-dimensional boundary layer, the velocity component in the direction of propagation of the disturbance may be regarded as a two-dimensional flow for stability purposes, and the usual stability theory applied. In addition he was able to demonstrate that, in the limit of infinitely large Reynolds number, neutral disturbances of the stationary-vortex type are possible for profiles having a point of inflection which coincides with a point of zero velocity. Near the leading edge of a yawed cylinder this particular profile lies in a plane which never deviates from the cross-flow direction by more than 6.5° – a result that is entirely consistent with the observations from the sublimation patterns. A similar set of conclusions was produced independently by Owen & Randall (1953). However, since the theory was valid only for infinite Reynolds number and the assumptions of negligible surface and streamline curvature could not be formally justified, this early theoretical work could only be interpreted in a qualitative sense.

Since the pioneering work of the early 1950s there have been several other investigations. Allen & Burrows (1956) and Burrows (1956) conducted measurements in flight on the boundary layer formed on untapered swept wings. Both tests produced results which were in qualitative agreement with the Owen & Randall (1952) correlation, but the difficulties associated with flight testing prohibited satisfactory quantitative comparisons. A comprehensive wind-tunnel investigation of the effects of yaw on transition on an aerofoil was carried out by Boltz, Kenyon & Allen (1960) in the 12 ft pressure wind tunnel at NASA Ames. However, the boundary-layer computations performed in support of the measurements were based upon an approximate (Polhausen) method, and so, once again, only qualitative conclusions could be drawn. Nevertheless, the destabilizing effect of yaw was observed, as was the appearance of vortrex traces in the transition visualization pattern (biphenyl crystals), and the estimated values of the cross-flow Reynolds number at transition were found to be in the range 190–260. Although these values were somethat higher than those quoted by Owen & Randall, the authors were apparently satisfied that this was due to computational deficiencies and also differences in the measuring techniques. At about this time considerable effort was also being directed towards the development of wings on which laminar flow was to be maintained by the use of surface suction. Almost all the work was documented in commerical reports, but comprehensive summaries may be found in Lachmann (1961) and, more recently, Pfenninger (1977) and Edwards (1977). In the present context, significant progress was made in two particular areas. The first was the development of numerical techniques for the solution of the laminar-flow stability problem with some of the effects of viscosity included. This was accomplished by Brown (1961). Brown's computations were extensions of Stuart's (1952) analytical work, and they demonstrated that, in the absence of all curvature and non-parallel flow effects, the minimum critical Reynolds number for cross-flow velocity profiles was of order 100 with no surface suction and that the critical Reynolds number rose very rapidly when suction was applied. The second was the discovery that the spanwise flow could be 'contaminated' by disturbances placed on the leading edge. Considerations based upon small-disturbance stability theory suggest that, for a fixed yaw angle, the location of the transition front will move progressively towards the attachment line, or 'stagnation' line, as the free-stream Reynolds number is increased. Consequently, the attachment line ($x = 0$) is the last location to develop an instability. This instability occurs when the spanwise momentum-thickness Reynolds number R_θ exceeds approximately 230 (Poll 1979). However, this argument is invalid if the attachment-line flow is subjected to large disturbances. It was found in flight tests (Gaster 1967) that disturbances generated in the wing/fuselage junction, or even at relatively small devices such as boundary-layer fences, were sufficiently large to produce turbulent flow at the attachment line when the spanwise momentum-thickness Reynolds number exceeded 100. Therefore turbulent flow was occurring at a unit Reynolds number that was only $\frac{1}{5}$ of the expected value. The implication of this discovery on the work that had gone before will be considered in §3.

3. Shortcomings of the early work

It appears that up until the discovery of the 'attachment-line contamination' mechanism the Owen & Randall criteria of 'constant cross-flow Reynolds number' for instability and transition on a yawed cylinder were generally accepted. However, the Owen & Randall criteria were based upon the results of the Anscombe & Illingworth (1952) experiment in which measurements were made on a swept wing that passed through the floor of a wind tunnel with a closed test section. Clearly, the junction between the wing and the tunnel floor would be expected to act as a source of large disturbance for the attachment-line flow, in much the same way as the wing/fuselage did in the flight tests on the laminar-flow wings. Therefore the Owen & Randall criteria should be reconsidered. For the wing section under test Owen & Randall calculated that the maximum value of χ was reached at about 5% chord. The combinations of tunnel speed and sweep angle necessary for the observation of striations (vortices) in the laminar flow and also for the observation of transition at this location, as reported by Anscombe & Illingworth, are presented in figure 3. The figure also contains the lines of χ_{max} equal to 125 and 175 and a line corresponding to conditions at which the spanwise R_θ at the attachment line is equal to 100. It is clear that, whilst conditions necessary for the appearance of stationary disturbances within the laminar flow are well described by a constant value of χ_{max}, the conditions necessary for the observation of transition are not. Instead the transition behaviour is in almost perfect agreement with the criterion for attachment-line contamination. Examination of the experimental configurations used by Allen & Burrows (1956), Burrows (1956) and Boltz *et al.* (1960) suggests that in all cases attachment-line contamination would have been possible for spanwise momentum-thickness Reynolds numbers greater than 100, and it should be noted that each of the above authors

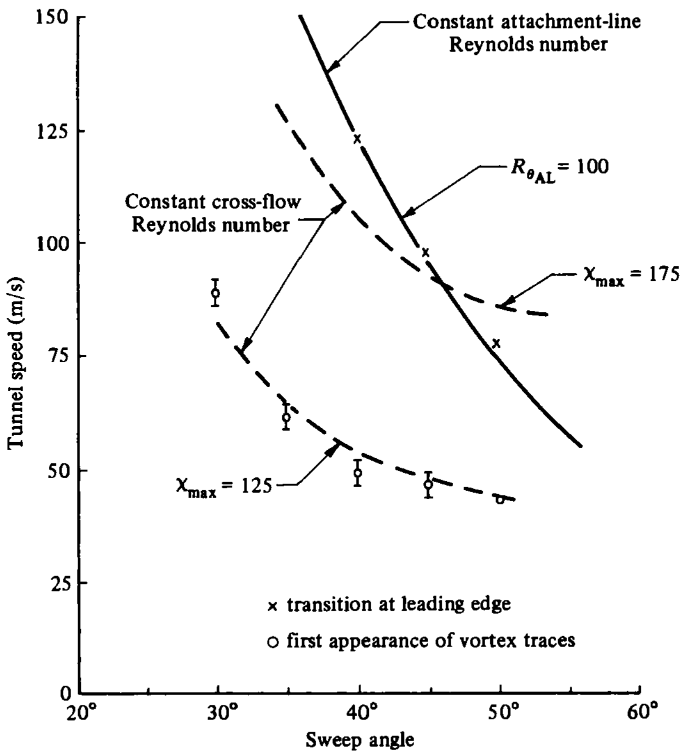


FIGURE 3. A comparison between the experimental results of Anscombe & Illingworth and criteria for cross-flow instability and attachment-line contamination.

claimed substantial agreement between their results and the Owen & Randall criterion. Therefore, in view of this, the constant χ_{max} criterion for transition via cross-flow instability must be viewed with some suspicion.

With regard to the theoretical work performed to date, the emphasis has been placed upon finding supporting qualitative arguments for the scant experimental data. The stability computations have demonstrated the low critical Reynolds numbers for cross-flow profiles and the existence of profiles that are unstable to disturbances that are stationary relative to the surface. However, these results cannot be used to predict the onset of transition, and it seems likely that, before progress can be made in this area, the effects of body and streamline curvature will have to be included in the stability equations.

4. The present experiment

The object of the present work was to obtain accurate information on the process of boundary-layer transition on the windward face of a yawed cylinder. To achieve this the experiment was designed with five specific points in mind. First, the cross-flow Reynolds numbers generated should be as large as possible and, in any event, significantly larger than those obtained in previous investigations. Secondly, in order to determine parametric trends accurately, the range of cross-flow Reynolds number should be as wide as possible. Thirdly, there should be no premature transition through contamination of the attachment-line flow. Fourthly, whenever practical, transition should be detected by several independent techniques. Finally, in order to provide some guidance for future theoretical exercises, an attempt should be made to determine the nature of the instability of the laminar flow immediately ahead of the transition location.

As a first step the boundary-layer development near the leading edge of an infinite yawed ellipse was considered. Preliminary calculations, based upon the approximate solutions given by Rosenhead (1963), revealed that the cross-flow Reynolds number χ reaches a maximum close to the leading edge such that

$$\chi_{\max} = A\left(\frac{Q_\infty r}{\nu}\right)^{\frac{1}{2}},$$

where Q_∞ is the free-stream speed, r is the leading-edge radius and A is a function of the yaw angle Λ and the thickness-to-chord ratio t/C. Moreover the function A was found to be only weakly dependent upon t/C, for ratios lying between 0 and 1, but strongly dependent upon yaw, with a maximum occurring at an angle of approximately 60°. Therefore, since a large leading-edge radius was clearly necessary and no particular restrictions were being placed upon the thickness-to-chord ratio, it was decided that the circular cylinder would be the most appropriate test model. This shape had the added advantage of being relatively simple to construct. However, in the present case, it was not necessary to manufacture a new model, since an adequate model was already available. This was a brass pipe with a radius of 114 mm which was faired from a point just behind the maximum-thickness location to form a 'teardrop' section with a 50% thickness-to-chord ratio. The fairing shape was designed to give continuity of surface slope and curvature at the blending point with the cylinder ($x/C = 0.46$). A general-arrangement drawing of the model mounted at 60° to the free-stream flow in the College of Aeronautics 2.4 m × 1.8 m low-speed wind tunnel is given in figure 4. This model was originally designed by Cumpsty & Head (1969) for an investigation of turbulent attachment-line flow, and it had also been used by Poll (1979) to study transition at the attachment line. Consequently, the attachment-line contamination problem which had gone unrecognized in some of the early work was already well understood for this particular configuration.

The model was designed for a nominal yaw angle of 60°, but in order to investigate the effects of yaw separately the model was pivoted at the upstream end to enable the angle to be varied from 55° to 70°. The wind tunnel was capable of providing free-stream unit Reynolds numbers in the range $0.5 \times 10^6\ \mathrm{m}^{-1}$–$3.7 \times 10^6\ \mathrm{m}^{-1}$, and the flow quality was such that the free-stream turbulence level Tu was always less than 0.16%. This parameter range was sufficiently broad for the establishment of boundary layers in which $\chi_{\max}$ could be as high as 350. This is almost twice the maximum value obtained in the Anscombe & Illingworth experiment and 1.4 times that achieved in the Boltz *et al.* tests. Since the experiment was conducted at high Reynolds number the laminar boundary layer on the windward face of the cylinder was very thin, being typically less than 1 mm. This meant that the boundary-layer characteristics could not be measured accurately by conventional techniques such as hot-wire or Pitot traverse. Therefore those flow parameters used in the analysis of the data were computed from the surface-pressure distribution by using the incompressible boundary-layer code developed by Beasley (1973).

Regarding the detection of instability and transition, all the measurements were made along a line parallel to, and 2 cm upstream of, the midspan row of pressure tappings as indicated in figure 4. Four different and independent techniques were employed. Two visualization methods were used. The first was the china-clay

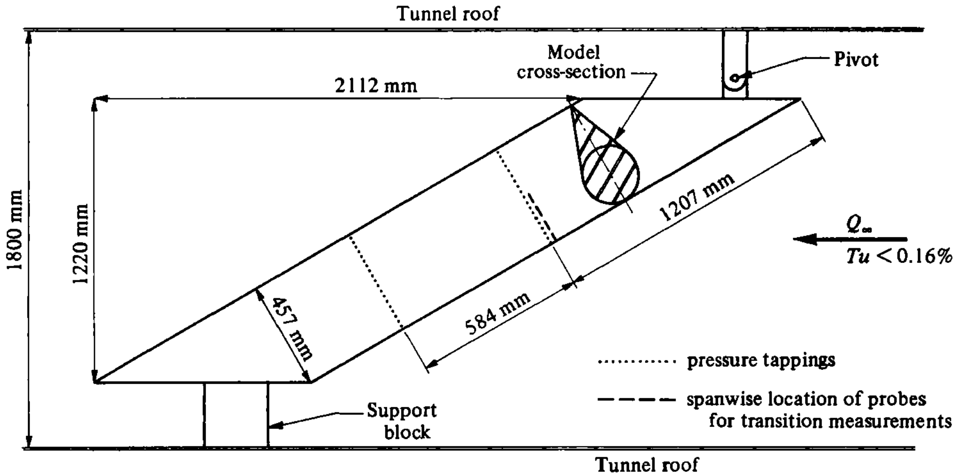


FIGURE 4. Experimental arrangement of the yawed-cylinder model in the College of Aeronautics 2.4 m × 1.8 m low-speed wind tunnel.

evaporation technique originally developed by Richards & Burstall (1945). This is the 'classical' method for the detection of cross-flow instability, since it reveals the streamwise striations and the characteristic 'sawtooth' transition front. The second was the simple surface oil-flow technique using a conventional suspension of titanium dioxide powder in kerosene as described by Maltby & Keating (1962). This method of visualization had not previously been used in connection with transition on a yawed cylinder. In order to obtain additional information on the nature of the instability a single miniature hot-wire probe was introduced into the boundary layer. This was supported approximately 0.25 mm from the surface in a cradle which was attached rigidly to the model. In this way all relative motion between the probe and the surface was eliminated. A second hot-wire probe was positioned on the attachment line ($x = 0$) to monitor the state of the spanwise boundary layer. This was to ensure that attachment-line instability and transition could not be confused with any other mechanisms which may be present. The bridge output voltages were displayed on a dual-beam oscilloscope. The fourth device used to detect transition was the surface Pitot tube. Two 1 mm diameter steel hypodermic tubes were flattened to give aperture heights of 0.125 mm and 0.250 mm. These were placed at the desired locations on the model and aligned with the local wall shear-stress vector, as given by surface oil-flow patterns. The conditions necessary for the onset and end of boundary-layer transition were then determined by observing the variation of indicated local dynamic pressure q_L with free-stream dynamic pressure q_∞ and noting those values of q_∞ at which the characteristic slope discontinuities occurred. A typical example of the variation of q_L with q_∞ is given in figure 5.

5. Preliminary considerations

Before any transition measurements were made the surface-pressure distribution was measured at the two spanwise stations indicated in figure 4 over the full range of yaw angle. A summary of the results is given in figure 6. For a fixed x/C it was found that, at all yaw angles, there was a small pressure gradient in the spanwise direction y. The maximum pressure difference between the sets of tappings was equivalent to 2.9% of the free-stream dynamic head. Cumpsty & Head also

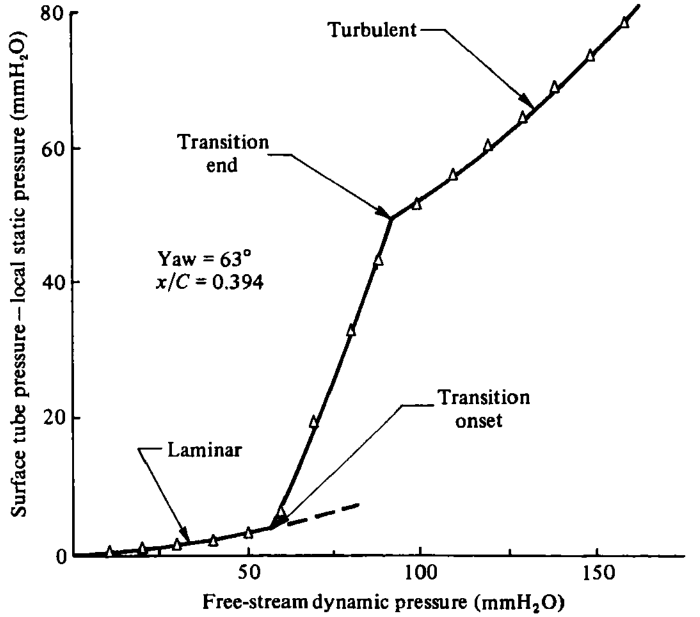


FIGURE 5. An example of the variation of the dynamic pressure indicated by a surface Pitot tube with free-stream dynamic pressure.

encountered a similar effect in their tests where the difference was 4.4 % of the free-stream dynamic head. Doubts concerning the effect of this pressure gradient on the velocity profiles had previously arisen in the context of the attachment-line investigation conducted by Poll (1979). As a result laminar attachment-line velocity profiles had been measured at two stations 1.3 m apart for yaw angles of 55° and 70° with a free-stream Reynolds number of approximately 2×10^5 ($\delta_{0.995}\approx 2$ mm). In addition Cumpsty (1977) had measured turbulent velocity profiles at two stations 0.9 m apart for a yaw angle of 60° and a Reynolds number of 7.2×10^5 ($\delta_{0.995}\approx 2.5$ mm). The results indicated that there was no significant spanwise variation, and, for the laminar profiles, there was excellent agreement between the data and the theoretical velocity distributions. Since the spanwise pressure difference was greatest on the attachment line ($x = 0$), it was concluded that the flow over the midportion of the wing was indeed a very close approximation to the infinite-yawed condition.

The boundary-layer prediction method required the exernal flow field to be supplied in the form

$$\frac{U_e}{U_\infty}=\frac{U_e}{U_\infty}(x/C),\quad \frac{V_e}{V_\infty}=\text{constant}.$$

Therefore the pressures were converted to velocity data by using Bernoulli's equation and smoothed by fitting a seventh-order polynomial containing only odd powers in x/C, i.e.

$$\frac{U_e}{U_\infty}=a_1\frac{x}{C}+a_3\left(\frac{x}{C}\right)^3+a_5\left(\frac{x}{C}\right)^5+a_7\left(\frac{x}{C}\right)^7.$$

The coefficients were determined by requiring that the sum of the squares of the errors be a minimum. It was found that over the range of yaw angles considered the chordwise velocity distributions could be adequately represented by three versions of the polynomial. The coefficients are given in table 1.

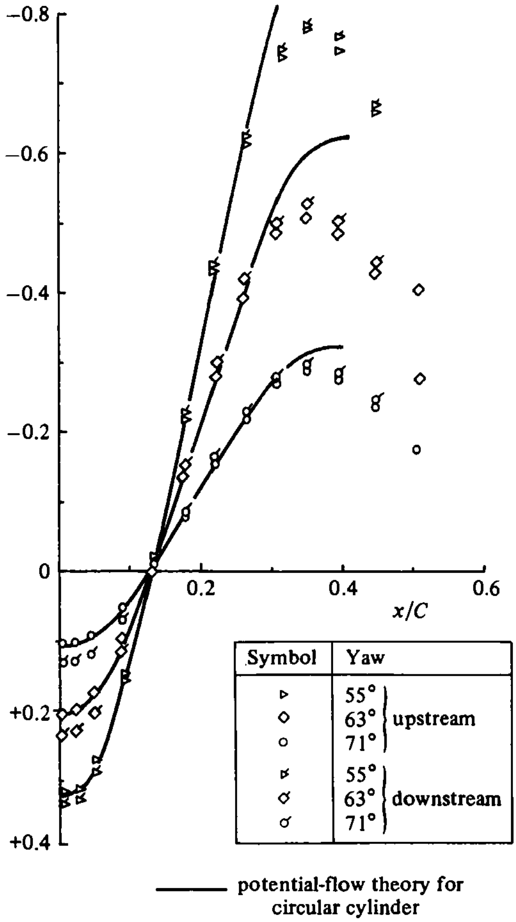


FIGURE 6. The surface-pressure distribution for the yawed-cylinder model.

Yaw range	a_1	a_3	a_5	a_7
52.5°–57.5°	+7.755	−16.90	−52.0	+196
57.5°–65.0°	+8.015	−21.75	−16.7	+113
65.0°–72.0°	+8.290	−24.05	−0.2	+64

TABLE 1

6. Results

Since previous investigators had obtained transition information by using surface-evaporation techniques, it was decided that 'china-clay' visualization would be tried first. The model was covered with a thin layer of kaolin using the water-based method described by Richards & Burstall (1945). This coating was white when dry. However, when the model was sprayed with the methyl salicylate indicator the kaolin became transparent and the black surface of the model showed through. During a wind-tunnel test the rate of evaporation of the indicator depends upon the state of the boundary layer. A turbulent flow causes the kaolin to dry out much more rapidly than a laminar flow for the same free-stream conditions. Since the change in evaporation rate is

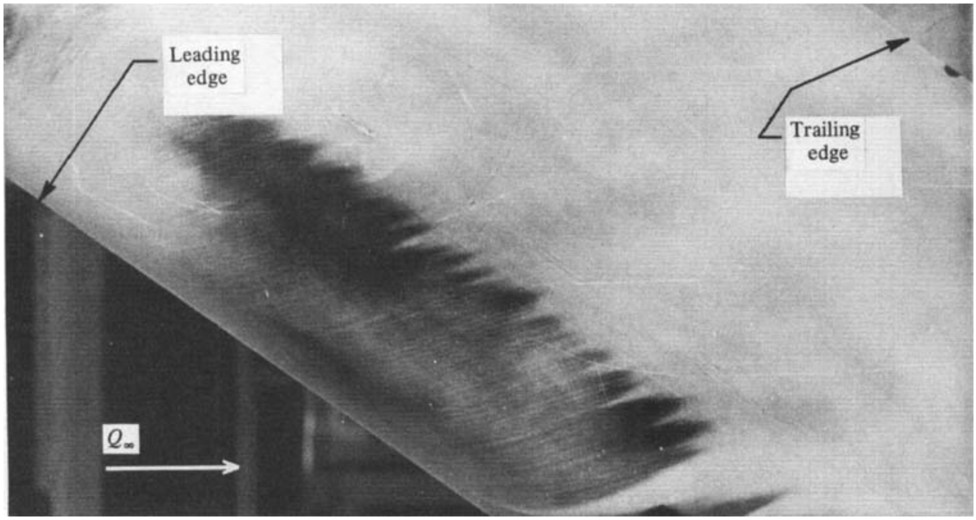


FIGURE 7. A china-clay evaporation visualization of the transition process on a cylinder. The yaw angle is 55° and the free-stream unit Reynolds number is 1.91×10^6 m^{-1}. Note the characteristic streaks in the laminar flow region ahead of the 'sawtooth' transition front.

almost discontinuous at the transition front, this feature appears as a sharp dividing line between black and white regions. The process typically takes between three and five minutes when the free-stream speed is of order 30 m/s. With the model yawed at 55°, tests at low speed showed that the flow over the windward face of the model supported an attached laminar boundary layer. This laminar layer separated on the leeward side of the model ($x/C \approx 0.41$), with subsequent turbulent reattachment to form a short bubble approximately 3 cm wide. When the free-stream unit Reynolds number was increased to 1.48×10^6 m^{-1} regular streamwise streaks with a pitch of about 4 mm were observed in the laminar region. These streaks appeared to originate approximately 8 cm back from the attachment line ($x/C \approx 0.18$), and were visible right up to the separation line. At a unit Reynolds number of 1.91×10^6 m^{-1} the streaks began at about 6 cm ($x/C = 0.13$) and the laminar separation gave way to attached transition. This visualization pattern is shown in figure 7. The transition front is sharply defined and it has a distinct sawtooth appearance. In addition, the pattern of closely spaced streamwise streaks on the windward face is clearly visible.

Whilst the china-clay method proved adequate for preliminary tests and qualitative comparisons with previous work, the kaolin coating was very fragile and began cracking and flaking off after only a few indications had been obtained. The kaolin was therefore completely removed and the surface-oil-flow technique was adopted. This involved brushing a suspension of titanium dioxide in kerosene on the windward surface of the cylinder before the wind-tunnel was started. With the wind-tunnel at test speed the oil flowed back over the surface to form a very thin film. After approximately five to ten minutes at typical test speeds the kerosene had completely evaporated and the titanium dioxide was left behind, forming a powdery coating on the model. It was found that this coating contained patterns which were very similar to those produced by the china-clay evaporation. An example is given in figure 8. In this case the yaw angle is again 55° and the free-stream unit Reynolds number is 1.89×10^6 m^{-1}. The oil has flowed back until the laminar separation line is encountered at 18 cm from the attachment line ($x/C = 0.40$). Ahead of the separation

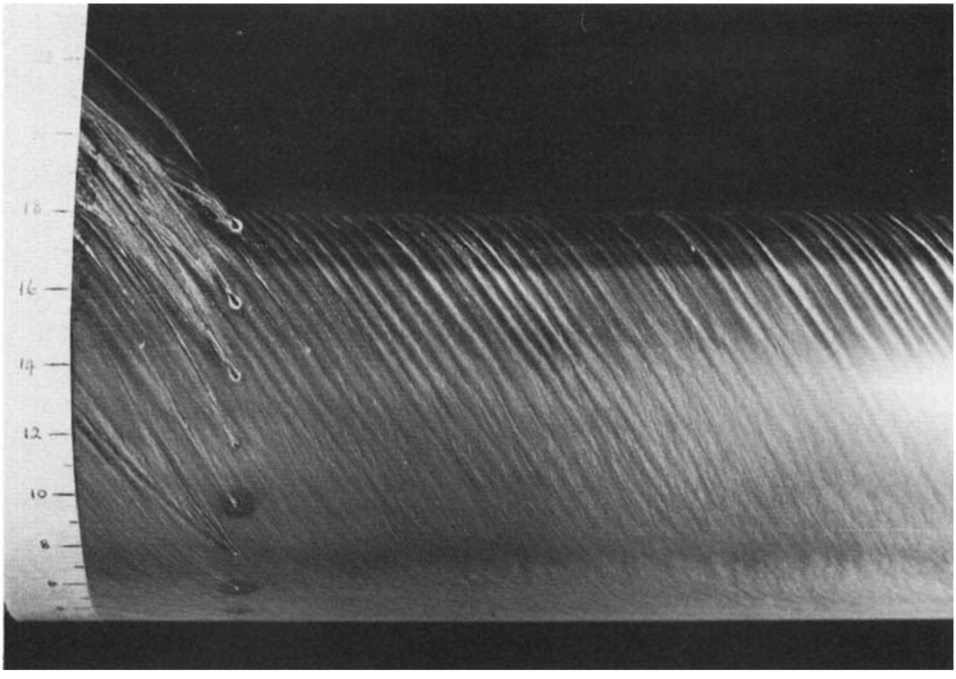


FIGURE 8. A close-up view of the stationary perturbations in the unstable boundary layer upstream of a laminar separation line. The pattern has been produced by the surface-oil-flow method with a yaw angle of 55° and a free-stream unit Reynolds number of $1.89 \times 10^6\ m^{-1}$. Numbers along the left-hand side of the picture indicate the distance from the attachment line measure in centimetres.

line, clear, evenly spaced, almost-streamwise streaks are visible. The origin of the streaks appears to be approximately 12 cm from the attachment line, and the pitch is between 3 and 4 mm. Figure 9 shows the flow pattern at the same yaw angle but at the higher unit Reynolds number of $2.31 \times 10^6\ m^{-1}$. At this condition the laminar separation line has been suppressed by a transition of the attached boundary layer. The turbulent flow begins at about 20 cm and leaves an almost-uniform coating of titanium dioxide powder. Once again the laminar region contains the streamwise streaks, but this time they originate about 8 cm from the attachment line. There is evidently a strong superficial resemblance between the visualizations presented in figures 9 and 7.

The oil-flow technique was found not only to give much clearer visualizations than the china-clay evaporation method but also to provide repeatable quantitative results. Since the final pattern consisted of a coating of dry powder, it was possible to remove a section from the model by the use of transparent adhesive tape and transfer it to a flat surface. This produced a permanent record which could be analysed accurately. The conditions necessary for the appearance of the streaks, the variation of the pitch of the streaks and the orientation of the streaks relative to the chordwise surface coordinate x for three different values of the yaw angle were obtained in this way. The results are presented in figures 10–12.

Visualization techniques like china-clay evaporation and surface oil flow can produce patterns which indicate the presence of those disturbances which are stationary relative to the surface. Since the patterns develop over a long time period it is clear that travelling disturbances could not affect the result, even if their amplitudes were large. In order to examine the travelling disturbances in the laminar

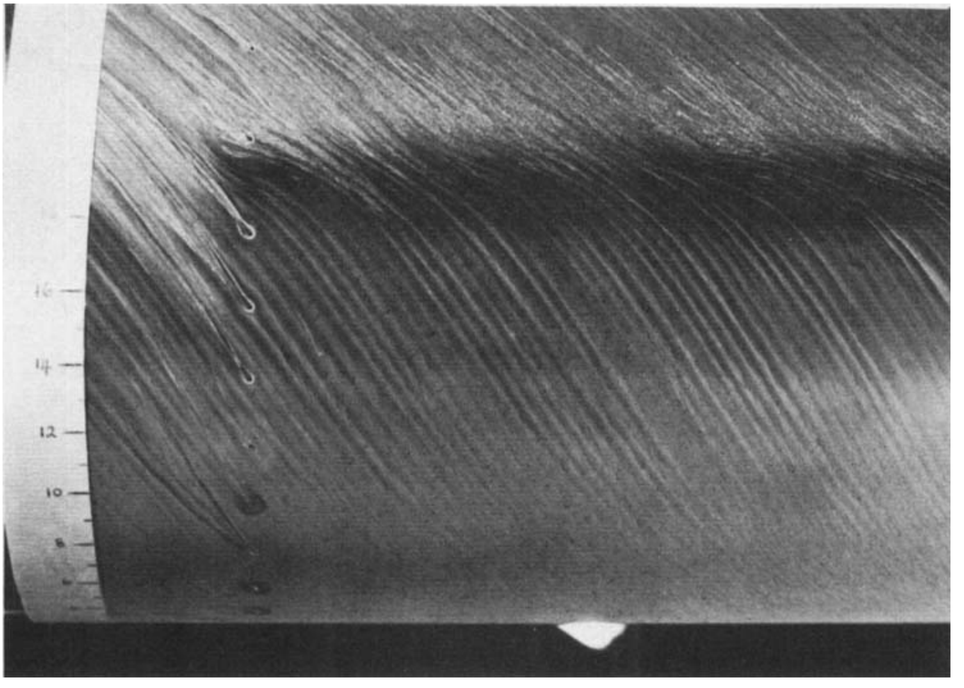


FIGURE 9. A surface oil-flow visualization of transition in the attached boundary-layer flow. The yaw angle is 55° and the free-stream unit Reynolds number is $2.31 \times 10^6\ \text{m}^{-1}$. Transition takes place at approximately 20 cm from the attachment line. Note the similarity between this pattern and that presented in figure 7.

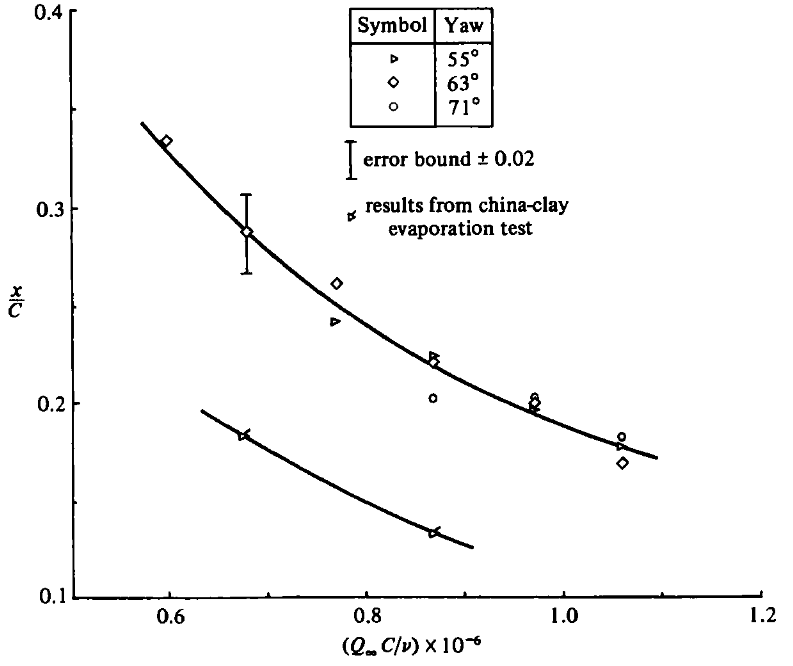


FIGURE 10. Conditions necessary for the first appearance of the streaks in the oil-flow patterns.

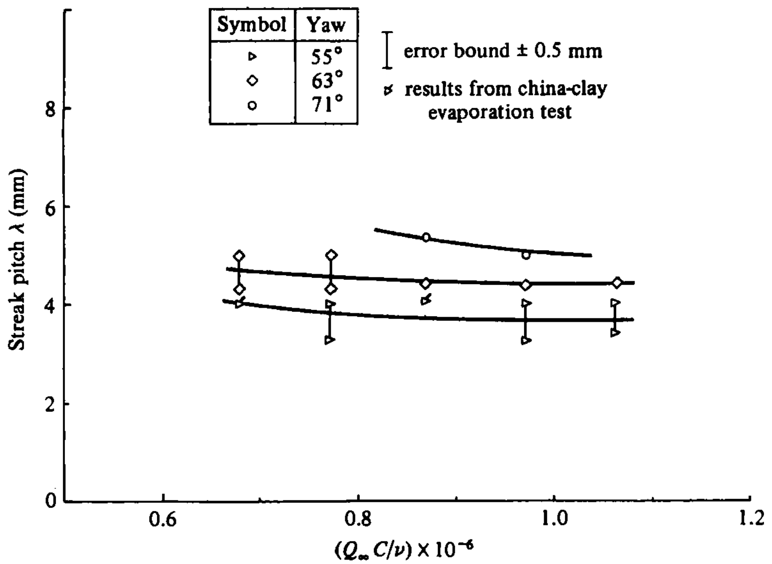


FIGURE 11. The variation of streak pitch with Reynolds number and yaw angle for x/C lying between 0.26 and 0.38.

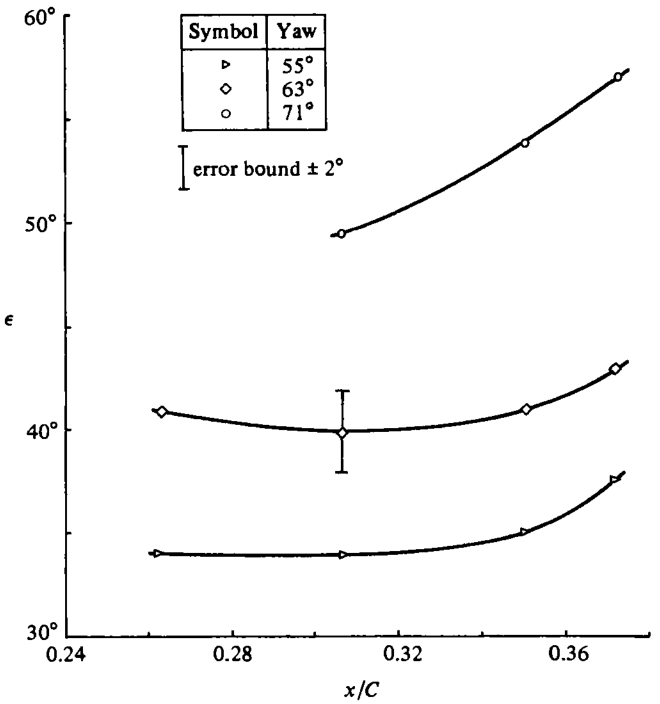
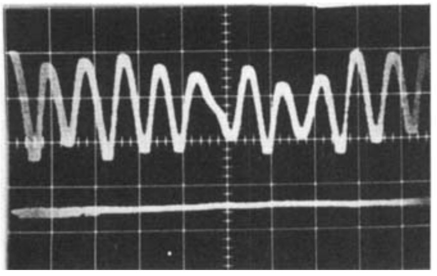
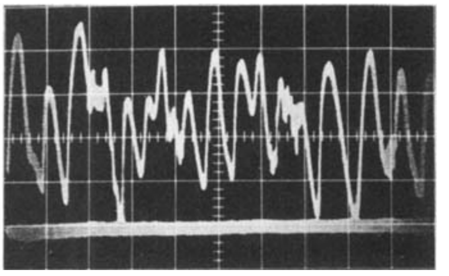


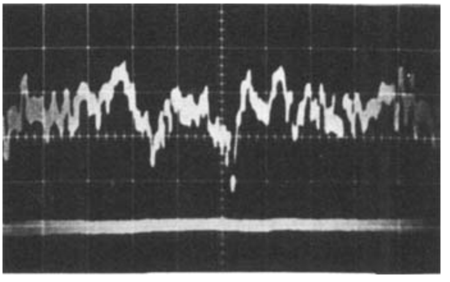
FIGURE 12. Orientation of the streaks relative to the normal-to-leading edge coordinate direction x.



(a)



(b)



(c)

FIGURE 13. Oscilloscope traces showing the velocity fluctuations at various stages in the development of flow instability when the cylinder is yawed at 63°. The upper trace shows conditions at $x/C = 0.306$, whilst the lower trace shows conditions at the attachment line ($x/C = 0$): (a) disturbed laminar flow $Q_\infty C/\nu = 0.90 \times 10^6$ ($\chi = 259$); (b) disturbed laminar flow 1.18×10^6 (297); (c) fully turbulent flow 1.65×10^6 (351). In each case $z = 0.25$ mm, the vertical scale is 0.1 V/large division and the time base is 1 ms/large division.

boundary layer, a single miniature hot wire was positioned 14 cm from the attachment line ($x/C = 0.306$). This lies in a region of negative pressure gradient and is the point at which the cross-flow Reynolds number exhibits a maximum for fixed free-stream Reynolds number. A second hot-wire probe was mounted on the attachment line to monitor disturbances in the spanwise flow. The tests were conducted at a fixed yaw angle of 63°. Figure 13 shows a sequence of results obtained by increasing the free-stream Reynolds number $Q_\infty C/\nu$ from 0.9×10^6 to 1.7×10^6. At a Reynolds number of 0.9×10^6 the boundary layer at an x/C of 0.306 was perturbed by a disturbance that had a fundamental frequency of about 1100 Hz ($f\nu/Q_\infty^2 \approx 1.8 \times 10^{-5}$). This is clearly visible in the upper trace of figure 13(a). At the same condition the lower trace shows that the attachment-line flow was stable. Increasing the Reynolds number to 1.2×10^6 caused the amplitude of the disturbance to increase and the basic frequency to shift to 1500 Hz. In addition a higher-frequency disturbance occasionally appeared superimposed or 'riding' on the fundamental wave. These high-frequency events are visible in figure 13(b), but are shown much more clearly in figure 14, where the high-frequency rider oscillates at 17500 Hz – this is one order of magnitude larger than the fundamental frequency. At the highest Reynolds number the signal at an x/C of 0.306 was completely turbulent as shown in figure 13(c). Even at this Reynolds number the lower trace shows that the attachment-line flow was both laminar and stable. Figure 15 gives an example of the signal obtained when the hot wire was placed just beyond the outer edge of the unstable laminar layer. This shows that the transition process involves the development

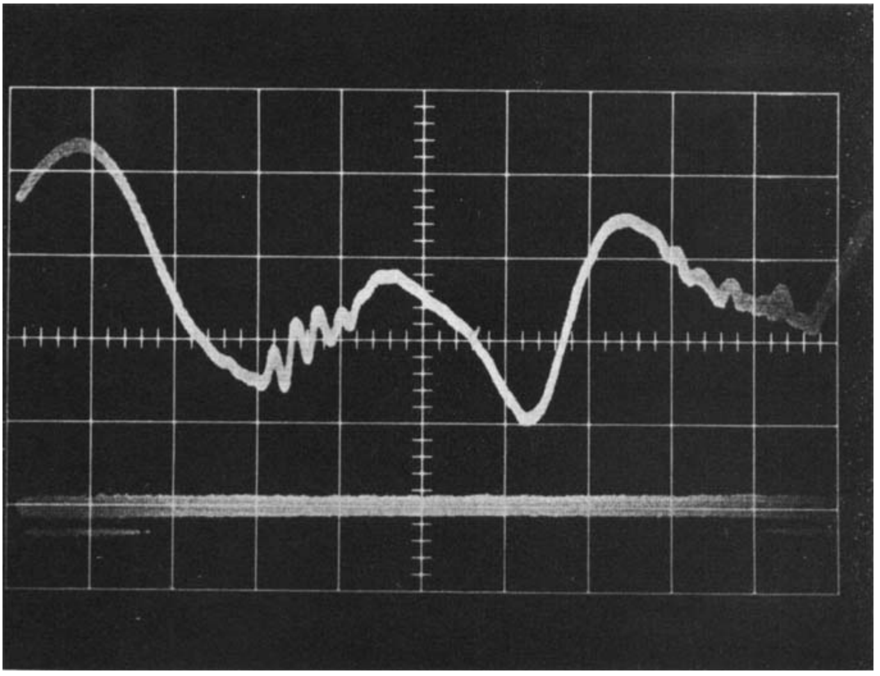


FIGURE 14. An example of a high-frequency rider superimposed upon the basic disturbance wave. Conditions are the same as those in figure 13(*b*). The vertical scale is 0.1 V/large division and the time base is 0.2 ms/large division.

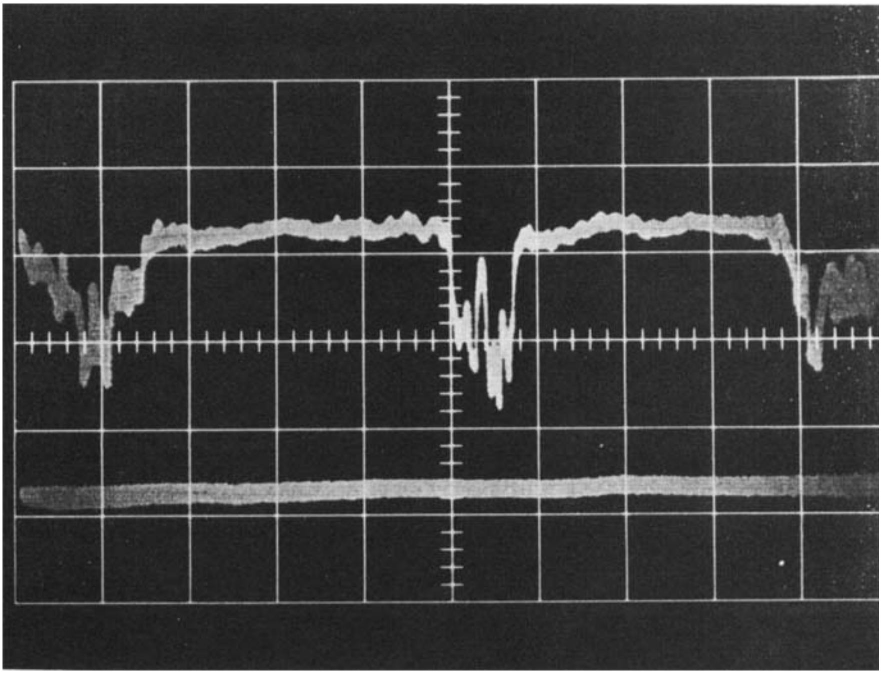


FIGURE 15. A trace showing the intermittent nature of the transitional flow in the outer part of the boundary layer. The hot wire is mounted 1 mm from the surface at $x/C = 0.306$. In this case the model is yawed at 63° and the free-stream Reynolds number $Q_\infty C/\nu = 1.43 \times 10^6$ ($\chi = 327$). The vertical scale is 0.1 V/large division and the time base is 0.5 ms/large division.

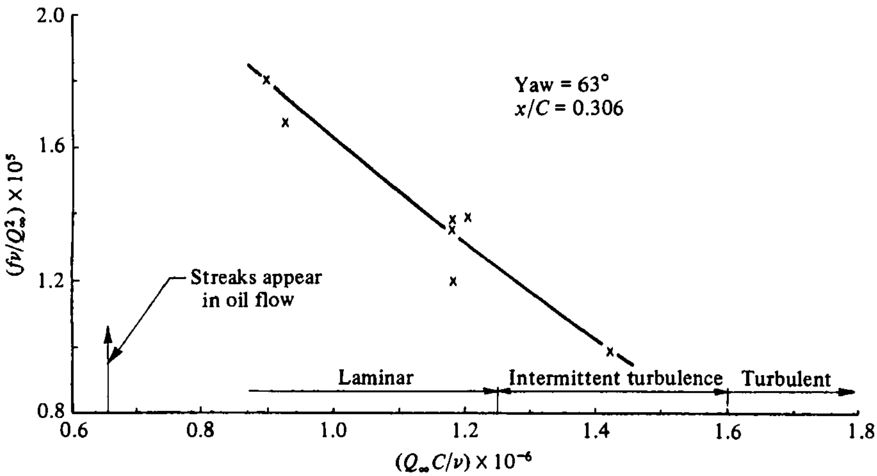


FIGURE 16. The dependence of the natural dominant frequency (in Hz) upon free-stream Reynolds number at fixed yaw and fixed chordwise location.

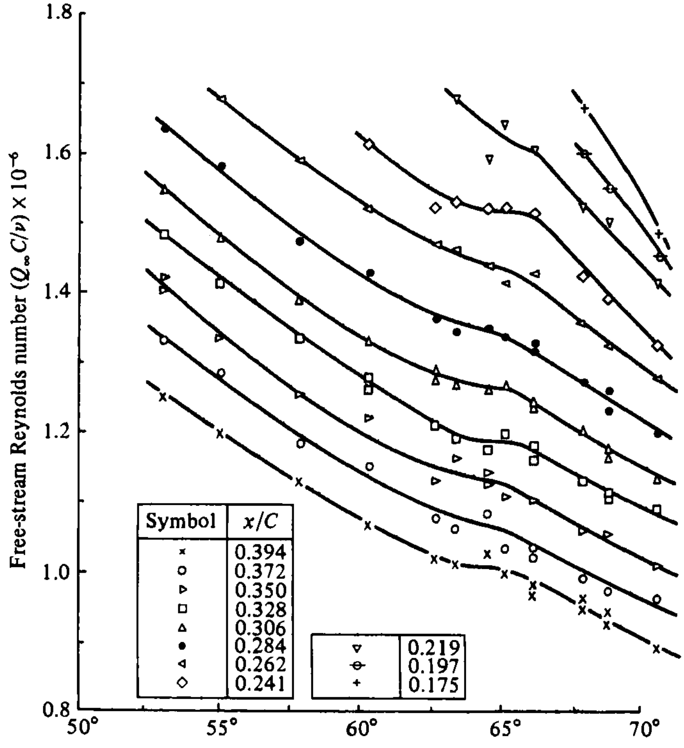


FIGURE 17. Variation of free-stream Reynolds number with chordwise position and yaw angle for the onset of transition.

of intermittent turbulence in much the same way as transition on the attachment line in the presence of roughness (Poll 1978), or transition in the boundary layer on a flat plate under conditions of zero pressure gradient. An indication of the variation of the observed 'natural' disturbance frequency f (Hz) with free-stream Reynolds number is given in a non-dimensional form in figure 16.

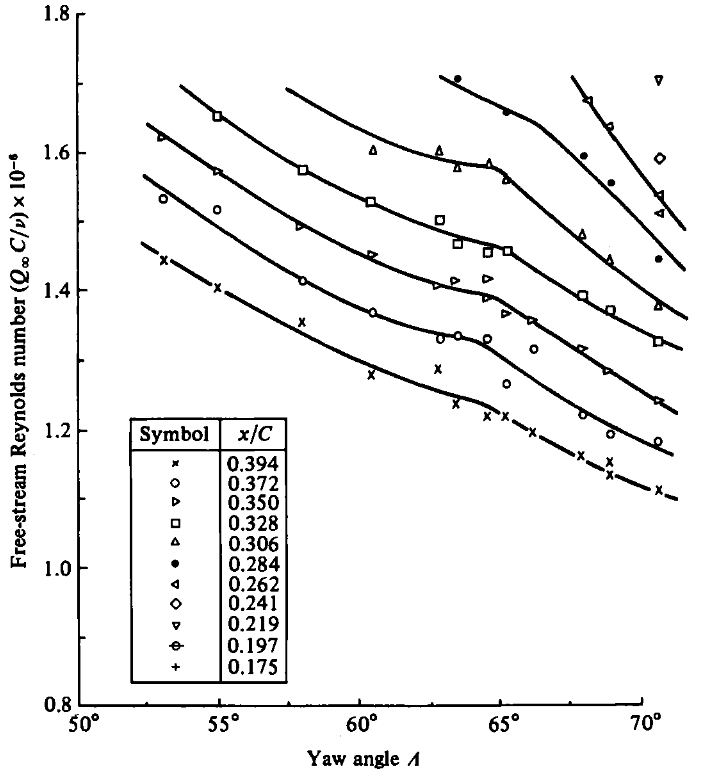


FIGURE 18. Variation of free-stream Reynolds number with chordwise position and yaw angle for the completion of transition.

The conditions necessary for the onset and completion of the transition process were largely determined by the surface Pitot-tube technique as previously described. Measurements were conducted at a fixed yaw and fixed chordwise location with tunnel dynamic pressure being the only variable. The results obtained for the onset and completion of transition are presented in figures 17 and 18 respectively. Both sets of data show very similar trends. In each case the critical free-stream Reynolds numbers were observed to decrease rapidly as yaw angle was increased, with a slight bump in the trends occurring between 63° and 66° depending upon location. It was found that, provided the air temperature in the tunnel was between 15 and 25 °C, the results were repeatable to within the scatter bands indicated on the figures. However, if the tunnel temperature was allowed to rise to 35 °C the free-stream Reynolds numbers for the onset of transition could be lowered by up to 6 %. This suggests that heat transfer between the model and the flow may also play a role in the transition process.

Finally, as a consistency check upon the different measurement techniques some simple comparative tests were performed. By mounting a Pitot tube and the hot-wire probe at the same location it was found that the Pitot-tube indication for the onset of transition coincided with the appearance of turbulent bursts in the hot-wire signal. Comparisons between the surface oil-flow indications and the Pitot tube showed that the oil-flow transition fronts (see e.g. figure 9) were in good agreement with the Pitot-tube transition-onset result. Therefore it appears that the presence of a thin layer of kerosene did not significantly affect the transition process. In the case of the china-clay evaporation technique, however, the (limited) data do suggest that, for the same yaw angle and free-stream conditions, the indicated transition fronts were closer to the attachment line than those obtained by the oil-flow or Pitot-tube methods. This discrepancy is probably due to the relatively rough surface finish of the kaolin coating.

7. Discussion

The surface visualization techniques produced patterns which contained streaks in the laminar flow ahead of the transition front. These patterns were qualitatively similar for both china-clay evaporation and surface oil flow and the general appearance was consistent with the observations of Gray and Anscombe & Illingworth. As indicated in figure 3, Owen & Randall found that the conditions necessary for the first appearance of streaks at a fixed surface location were such that the local value of a suitably defined cross-flow Reynolds number was constant. Therefore, as a first step, the data for the onset of streaks (figure 10) were correlated in the same manner. Since Owen & Randall did not provide a complete description of their cross-flow Reynolds number, it was decided that for the present purposes a suitable definition would be

$$\chi = -\frac{\bar{v}_{\max} \Delta}{\nu},$$

where $\bar{v}_{\max}$ is the maximum value of the cross-flow velocity and Δ is the height above the surface at which $\bar{v}$ drops to 1 % of its maximum value. The results are presented in figure 19. This figure clearly shows that the oil-flow data are collapsed, with the critical value of χ at approximately 220. The china-clay data also collapse but the value of χ is 170. We note that both these numbers are much larger than the Owen & Randall criterion, which, after suitable correction to conform to the new definition of χ, gives the critical value as 135. Therefore, for a given model geometry, the conditions necessary for the onset of streaks do correspond to a fixed value of χ. However, this value is not the same for all configurations. Moreover, at a given location the streaks appear in the china-clay evaporation patterns at a lower Reynolds number than is the case for the surface oil flow. This is not unexpected, since not only must there be differences in the surface texture, or roughness, associated with these methods but, more importantly, the evaporation responds to spatial variations in the magnitude of the wall shear stress whilst the oil-flow pattern is determined by the direction of the wall shear stress.

The early theoretical work of Stuart and Owen & Randall showed that at infinitely large Reynolds number the cross-flow velocity profile could not amplify stationary disturbances, but profiles for which a point of inflection coincided with a point of zero velocity could. This type is referred to as a 'critical profile'. Therefore the determination of the inclination of the stationary disturbances relative to the external flow direction yields important information concerning the laminar-flow stability. Consider the coordinate system sketched in figure 1. For the boundary-layer flow over the windward face of a yawed cylinder the components of the velocity in the ϵ direction have profiles of the form shown in figure 20. The profile for $\epsilon = \theta - \frac{1}{2}\pi$ is the cross-flow profile, and that for $\epsilon = \epsilon_{\mathrm{I}}$ is the critical profile. In figure 21 the orientation of the normals of the streaks is compared to that of the cross-flow profile, as determined from surface-pressure measurements, and also that of the critical profile, as determined from boundary-layer computations. The data indicate that the stationary disturbances are associated with a direction which is closer to the 'critical'

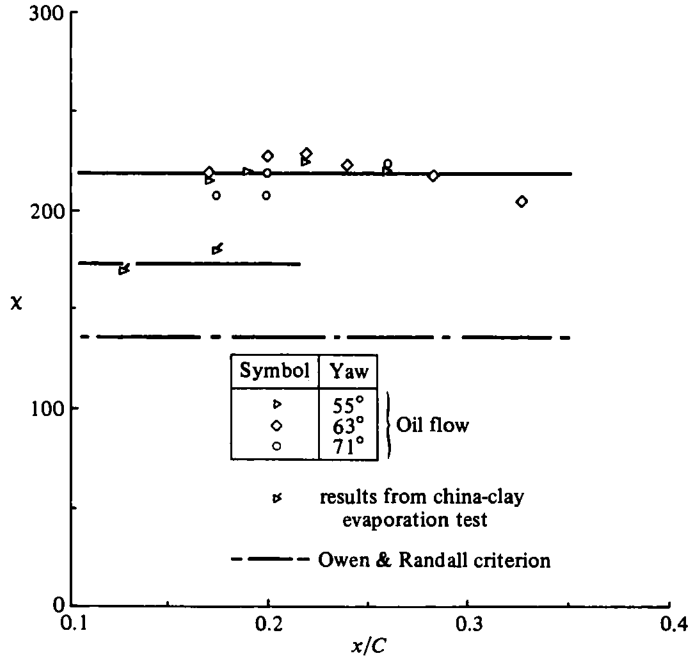


FIGURE 19. Variation of χ with chordwise position and yaw angle for the first appearance of streaks.

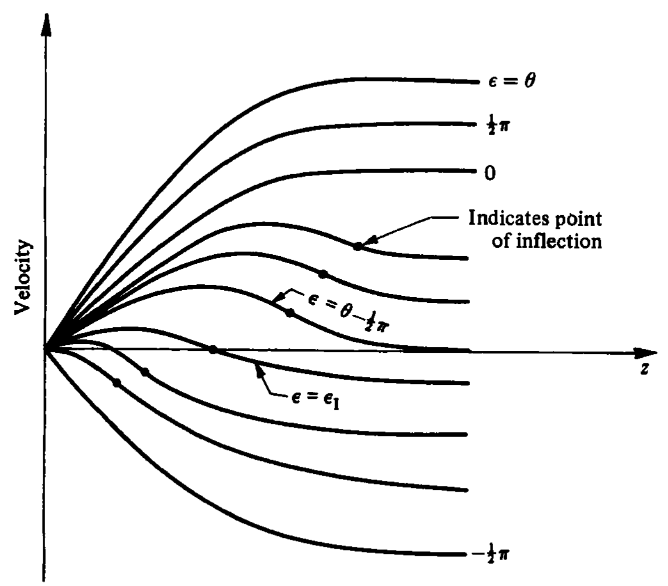


FIGURE 20. Variation of the velocity profile with disturbance propagation angle ϵ.

than the 'cross-flow'. At locations near the windward plane of symmetry the disturbance direction ϵ is consistently less than that of the 'critical', but with increasing chordwise distance the disturbance direction tends to approach that of the 'cross-flow'. This tendency to approach the cross-flow direction at large distances from the leading edge was also noted by Anscombe & Illingworth. It would appear then that the conditions necessary for the production of stationary disturbances are

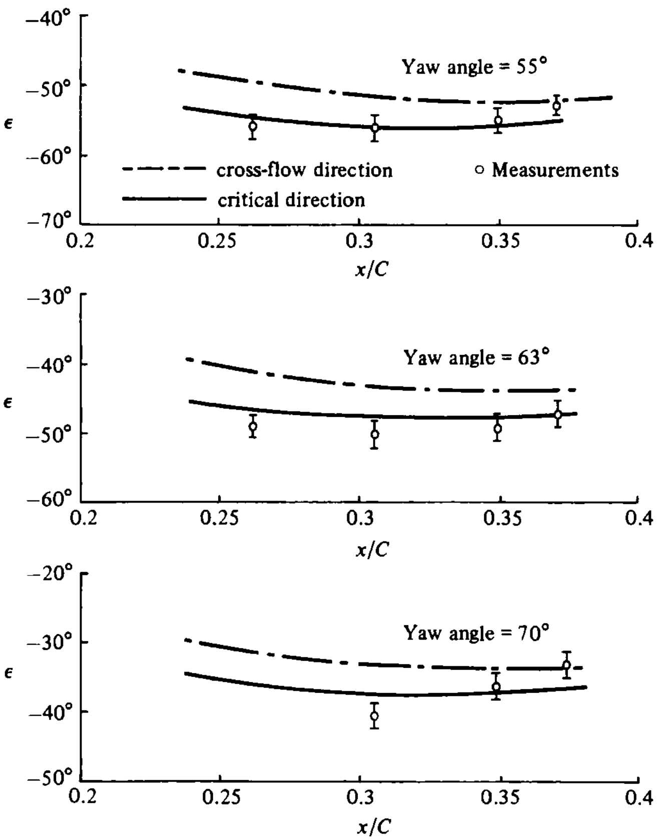


FIGURE 21. A comparison between the orientation of the streak normals and the cross-flow and critical profile directions.

determined by the stability of a velocity profile which is similar in appearance to the critical profile except that the inflection point occurs at a height z where the velocity is small but not zero. In view of this it is rather surprising that the conditions necessary for the appearance of the streaks are successfully correlated by a Reynolds number based entirely upon cross-flow profile parameters.

In the papers by Owen & Randall and Gregory *et al.* it was argued that the pitch of the streaks in a china-clay evaporation pattern was equal to one wavelength of the stationary disturbance. The visualizations conducted in the present study have shown that the pitch of the streaks is the same for both china-clay evaporation and surface oil flow. Therefore, assuming that the conclusions of the above authors are substantially correct, the data contained in figure 11 provide wavelength information. For the laminar boundary layer the characteristic viscous length-scale η is given by

$$\eta = \left(\frac{\nu x}{U_e}\right)^{\frac{1}{2}},$$

and all the various thickness scales can be related directly to this quantity. As indicated in figure 11, the measurements of the pitch λ of the streaks were made in the region between an x/C of 0.26 and 0.38 at free-stream Reynolds numbers ranging from 0.68×10^6 to 1.06×10^6. The pressure distributions indicated that over this region of the surface η was almost independent of x. This was because the flow was still accelerating, and therefore the growth rate of the viscous layer was small. However, at a fixed unit Reynolds number the viscous scale does depend strongly upon the yaw angle, and for this cylinder model it may be readily shown that

$$\frac{\eta}{C} \simeq \frac{0.43}{(Q_\infty C/\nu)^{\frac{1}{2}} \cos^{\frac{1}{2}} \Lambda}.$$

This variation is very similar to that of λ with yaw as shown in figure 11. Therefore, by using η as a normalizing variable, the disturbance-wavelength data are reduced to a single value of the parameter λ/η, i.e.

$$\frac{\lambda}{\eta} \approx 13.5 \pm 2 \quad \text{for } 0.26 < \frac{x}{C} < 0.38.$$

Whilst this expression conveniently describes the present observations, it should be pointed out that this is not a general result. In the Anscombe & Illingworth experiment it was found that the streak pitch was constant over large stretches of the chord for which the lengthscale η increased by at least a factor of two. This point was noted by Owen & Randall, who suggested that "as a rough general rule" the wavelength of the stationary disturbance should be three times the boundary-layer thickness at the point at which the cross-flow Reynolds number χ is a maximum. In terms of the viscous length η, the Owen & Randall rule becomes

$$\frac{\lambda}{\eta} \approx 12 \quad \text{at } \chi_{\max}.$$

We note that on the present model the maximum value of χ occurs at $x/C \approx 0.31$, and hence the results presented in figure 11 are in broad agreement with the Owen & Randall rule.

None of the early work, either experimental or theoretical, considered the relevance of time-dependent disturbances in the unstable laminar flow. Yet the three-dimensional boundary layer clearly provides for a wider variety of modes of instability than the more familiar two-dimensional flow. In a two-dimensional situation, such as flat-plate Blasius flow, it is found that, when the instability modes are triggered by 'natural' small-scale disturbances in the free stream, the instability is dominated by a single-frequency disturbance. This frequency is observed to vary with position and unit Reynolds number in some clearly defined and repeatable manner. An example of this behaviour is given by Schubauer & Skramstad (1947). Moreover, the observed variation of non-dimensional frequency with Reynolds number is found to lie entirely within the neutral loop, as determined by linear stability theory. Recently, a similar behaviour has been noted for the particular case of the attachment-line flow formed on a long inclined cylinder. This work has been reported by Hall, Malik & Poll (1984). Once again the variation of the dominant disturbance frequency with Reynolds number lies within the linear-stability neutral loop. The results presented in figure 16 are clearly consistent with the observations made in these less complex situations. A single dominant frequency appears, and this shifts as the unit Reynolds number varies. However, the data are not sufficiently extensive to define a lower Reynolds-number limit to the frequency function nor is any linear-stability neutral loop available for comparison. Nevertheless, several general statements may be made. In the first instance extrapolation of the results in figure 16 suggests that at the conditions necessary for the appearance of streaks in the oil-flow pattern the dominant natural frequency would have been approximately 600 Hz, whilst at the onset of transition the frequency was 1600 Hz.

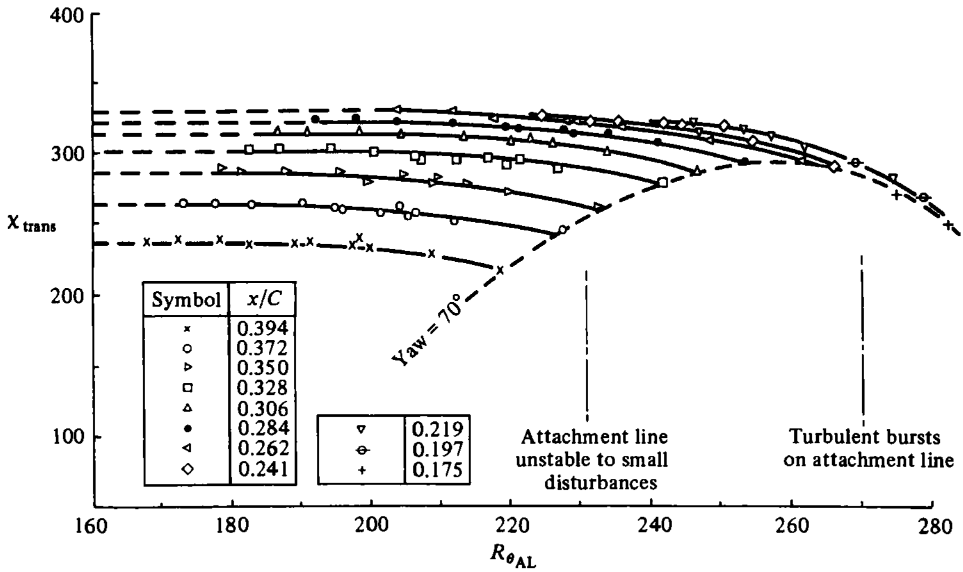


FIGURE 22. Variation of χ at transition onset with attachment-line momentum-thickness Reynolds number and chordwise position.

The hot-wire traces also indicate that the r.m.s. fluctuation for these disturbances are large. Estimates for the conditions depicted in figure 13 suggest that for case (*a*) the r.m.s. fluctuation is between 10 and 16 % of the local velocity, in case (*b*) this has risen to between 20 and 30 %, whilst in the completely turbulent case (*c*) the fluctuation level is only 9–12 %. This latter value is a typical fluctuation level for turbulent flow in the lower part of a boundary layer. Clearly, these results suggest that the boundary layer is capable of amplifying disturbances over a wide range of frequency to very large amplitudes. Therefore it is not at all certain that the observed transition process is in any way due to the stationary disturbance whose amplitude at transition is yet to be determined.

Regarding the conditions for the onset of transition it is appropriate once again to examine the 'χ = constant' hypothesis of Owen & Randall. The results presented in figure 17 are replotted in figure 22 as a variation of χ at transition with chordwise transition location and the attachment-line momentum-thickness Reynolds number $R_{\theta_{AL}}$. This presentation has several notable features. First, if conditions are such that the attachment-line flow is stable to small disturbances ($R_{\theta_{AL}} < 230$), then, at a given point on the surface, the cross-flow Reynolds number at transition onset is approximately constant. Secondly, the critical values of χ are considerably larger than the value of 200 proposed by Owen & Randall. In the present case χ_{trans} ranges from 240 to 325. This adds weight to the proposition that transition near the leading edge in the Anscombe & Illingworth experiment was the result of attachment-line contamination and not cross-flow instability as assumed by Owen & Randall. Thirdly, the critical value of χ seems to tend towards a constant value of about 325 as the transition front approaches the attachment line. It is possible to conclude that the 'χ = constant' hypothesis is successful in so far as it accounts for the effects of yaw angle and free-stream unit Reynolds number. However, in this form the present data indicate that at least two parameters are necessary for a complete description of the transition onset conditions.

In two-dimensional boundary layers it has been known for many years that transition criteria based upon a constant value of a characteristic Reynolds number, such as $R_{\delta*}$, are totally inadequate. The minimum number of parameters needed to provide an adequate (though not necessarily accurate) prediction for transition onset is two. A recent example of such a criterion is the R_x versus shape factor H relation developed by Wazzan, Gazley & Smith (1982). Given this situation, one would expect that, in general, the minimum number of parameters required to produce a successful transition-prediction method for three-dimensional flows would be three – one of which could obviously be the cross-flow Reynolds number.

The importance of shape factor in the two-dimensional problem strongly suggests that H may also be important in the three-dimensional case. In this context it is important to note that for flow on an infinite yawed cylinder the shape factors for the spanwise and chordwise velocity profiles depend only upon chordwise location. Therefore the critical cross-flow Reynolds numbers given in figure 22 could be expressed in terms of shape factor rather than chordwise position. This has the advantage that the results would be formulated entirely in terms of boundary-layer parameters rather than the geometric parameters of a particular cylinder shape. There is, however, an additional problem in that there is no uniquely defined shape factor that can characterize a general three-dimensional flow. It is therefore necessary to advance a hypothesis which can be subsequently tested against the experimental data.

For transition in an initially two-dimensional mean boundary-layer flow it is known that, even though the initial disturbances may be two-dimensional, the final breakdown process is always caused by three-dimensional disturbances. This breakdown process is still not completely understood, although many of the principal physical features have been described – see for example Klebanoff, Tidstrom & Sargent (1962) and more recently Saric & Thomas (1983). It appears that in the later stages the unstable laminar flow develops longitudinal vortices which cause the streamwise mean-flow profile to develop a spanwise variation. This variation is such that the mean-flow profile is subjected to alternating acceleration and deceleration close to the wall. When the deceleration becomes large enough the mean flow develops an embedded shear layer which produces a secondary, high-frequency, instability, and this leads to the production of turbulent spots. This being the case, the role of the shape factor becomes evident, since a mean profile with a small H-value has a larger variation of velocity near the wall, and consequently requires a much stronger perturbation to generate an embedded shear layer than would a profile with a large H. Therefore as a general rule the smaller the shape factor the larger the Reynolds number at transition. In the present problem the longitudinal disturbances do not have to develop from two-dimensional disturbances, since they are generated directly by the cross-flow instability and their strength may be judged from the magnitude of the cross-flow Reynolds number χ. Following a suggestion originally due to Arnal (1982) (see also Coustols 1983), we postulate that if the effect of the disturbances is to produce a secondary, embedded, shear-layer instability in the streamwise mean flow then the appropriate shape factor for correlation purposes would be the streamwise shape factor H_1. Some evidence for the 'shear-layer' hypothesis is provided by the hot-wire output shown in figure 14, where a high-frequency wave develops on top of an existing disturbance. The appearance of high-frequency travelling waves is characteristic of shear-layer instability. Therefore, provided that the instability of the flow is dominated by the cross-flow profile characteristics, we might expect that a flow with a low streamwise shape factor would require a large χ for transition and *vice versa*. It should be noted that if the streamwise

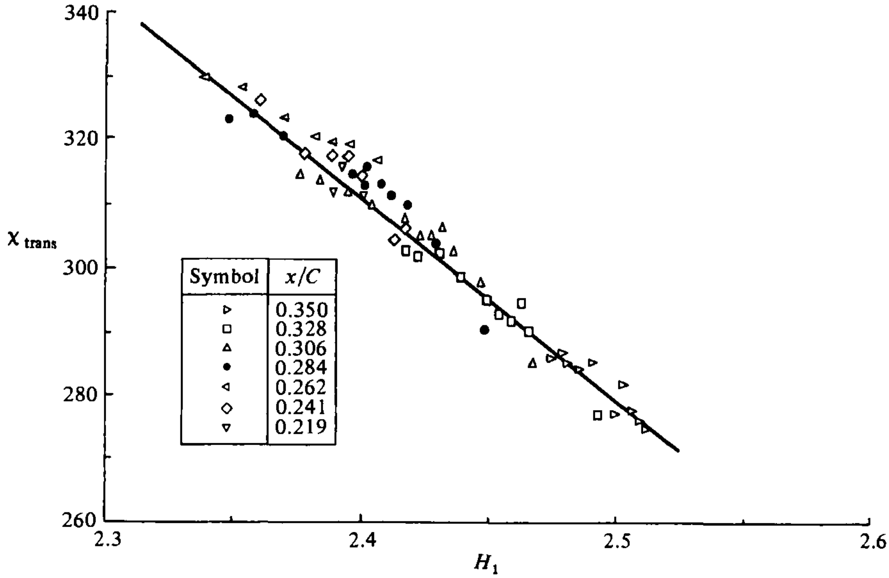


FIGURE 23. The variation of cross-flow Reynolds number with streamwise shape factor at the onset of transition. This plot contains only those conditions for which the streamwise flow profile is stable from $x = 0$ to $x = x_{trans}$.

flow is unstable at the transition point then, by analogy with the two-dimensional case, the conditions at transition must also be expected to depend upon a streamwise Reynolds number such as R_{δ_1}. A replotting of all the data contained in figure 17 in the form χ_{trans} versus H_1 produced only a limited collapsing of the data. This indicates that, in general, at least three boundary-layer flow parameters are needed to describe transition. However, when those data that, according to linear stability theory, had an unstable streamwise flow at the observed transition position, were excluded the correlation of χ_{trans} versus H_1 was excellent. This is given in figure 23.

8. Conclusions

The experiment has demonstrated that, at sufficiently high Reynolds number, yaw has a powerful destabilizing effect on the laminar flow over the windward face of a cylinder.

The instability manifests itself in two ways. First, there is the development of a fixed-wavelength disturbance which is stationary relative to the cylinder surface. This disturbance may be visualized by either the surface evaporation technique or the surface oil-flow technique, although at a given location the disturbances appear at a lower Reynolds number when the surface evaporation technique is used. The normals to the wave fronts lie close to, but are not perfectly aligned with, the plane in which an inflection point in the mean boundary-layer profile coincides with a point of zero velocity, i.e. the so-called 'critical' profile direction. Both visualization techniques indicate the same wavelength, and this is about thirteen times the characteristic viscous lengthscale at the point at which the disturbance appears. The conditions necessary for the appearance of the stationary waves may be correlated in terms of a suitably defined cross-flow Reynolds number. However, the critical value appears to vary with the cross-sectional shape of the cylinder. The second manifestation of the instability is one of large-amplitude time-harmonic waves of almost single frequency. The frequency is of the order 1 kHz, but at a fixed location this increases with increasing unit Reynolds number. Close to the surface the amplitude of these disturbances can exceed 20 % of the local mean-flow velocity. Prior to the breakdown to turbulence, the waves develop high-frequency riders. The frequency of these superimposed disturbances is approximately ten times that of the fundamental wave.

Following the laminar instability, transition begins with the appearance of intermittent turbulent bursts. The conditions necessary for the onset of transition cannot be correlated in terms of a single value of the cross-flow Reynolds number, as was previously proposed. In general, the results from the present experiment require at least three flow parameters to define the onset conditions adequately. However, by limiting the data set to those for which, according to linear stability theory, the streamwise velocity profiles were stable at the observed transition location, an excellent correlation was obtained when the cross-flow Reynolds number at transition onset was plotted against the local shape factor for the streamwise velocity profile.

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